Why Multi-Result Supercompilation Matters: Case Study of Reachability Problems for Transition Systems

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July 8, 2012
Third International Valentin Turchin Workshop on Metacomputation, META 2010
Pereslavl-Zalessky, Russia

History and Main Conclusion

- **2005-2007 Andrei Nemytykh and Alexei Lisitsa**
  - have experimentally found a method to solve the coverability problem for (a class of) practical counter systems (models of cache-coherence protocols and other systems) with the Refal Supercompiler SCP4
  - *User + single-result supercompiler = MRSC*

- **2010-2011 Andrei Klimov**
  - have theoretically explained and proved that the coverability problem is solvable for monotonic counter systems by an iterative procedure of applying a domain-specific supercompiler for counter systems varying a parameter of generalization
  - *An optimized MRSC enumerating a small subset of residual graphs*

- **2005-2007 Gilles Geeraerts *et al* (Belgium)**
  - theory of `Expand, Enlarge and Check' algorithmic schema (ECC) for solving the coverability problem of well-structured transition systems (WSTS)
  - *An MRSC for WSTS with reduced search space*

One thing to remember from this talk

- These are instances of domain-specific multi-result supercompilation (MRSC) with search space reduction based of domain properties and purpose

`User-controlled' MRSC

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- **Andrei Nemytykh devised two versions of SCP4**
  - $SCP4_0$ – standard version
  - $SCP4_1$ – generalization of empty expressions (representing zeros) prohibited

- **The user behavior**
  - When $SCP4_0$ did not prove the coverability, $SCP4_1$ was applied
  - No more supercompilers were needed for the considered samples borrowed from the collection by Giorgio Delzanno

- **Questions remained**
  - Were these SCP4 versions sufficient?
  - Might other restrictions of generalization be needed?
  - Had the SCP4 author to invent new modifications of SCP4?

Domain-Specific Special-Purpose MRSC

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- **Algorithm**
  - Scp_l – a domain-specific supercompiler for counter systems with parameter l prohibiting generalization of integers $n < l$
    - the simplest version: integers $n \geq l$ are immediately generalized
  - **for** l=1,2,3... **do**
    use Scp_l to build a residual set of configurations
    if all residual configurations are disjoint with the target set *Unsafe*
    then return "Unreachable"

- This algorithm with the simplest supercompiler Scp_l fits the ECC schema
- My proof of its correctness differs from that of the ECC algorithmic schema
- ...and asserts a stronger termination statement:
  - it terminates for all monotonic counter systems and upper-closed *Unsafe sets*

ECC as a domain-specific multi-result supercompiler

- **2005-2007 Gilles Geeraerts *et al***
  - theory of `Expand, Enlarge and Check' algorithmic schema (ECC) for solving the coverability problem of well-structured transition systems (WSTS)
  - *An MRSC for WSTS and its optimized versions*

- **Main ideas**
  - The set of all possible configurations $\mathbb{C}$ is infinite (as usual)
  - *Def*. A finite $R \subseteq \mathbb{C}$ is called a residual set *iff* it is closed under "driving":
    - Post($[\![R]\!]$) $\subseteq [\![R]\!]$
  - Consider an ascending sequence of finite sets of configurations C_l:
    - $C_0 \subset C_1 \subset C_2 \subset C_3 \ldots$
    - $\mathbb{C} = \cup C_l$
  - Consider residual sets $R \subseteq C_l$
  - The set $\{R \mid R \subseteq C_l\}$ of all such residual sets is finite as C_l is finite
  - Hence, it is solvable whether there exists a safe $R \subseteq C_l$ (that is, all configurations in R are disjoint with the target set $Unsafe$)
  - Iterate for l=0,1,2,3…
  - If a safe residual set $R \subseteq \mathbb{C}$ exists, then C_l s.t. $R \subseteq C_l$ exists and hence the iterative procedure terminates
- The notion of MRSC is wider

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Where is the well-structuredness of a TS used?

(WS = monotonicity + well-quasi-order)

- Existence of safe *R* when the TS is safe
- Optimizations: reducing the search space

Without the well-structuredness:

If there exists an inductive proof that a TS is safe with the inductive hypotheses in form of a residual set of configurations, then MRSC finds it

Related work: Supercompilation-like algorithms

1969 Richard M. Karp and Raymond E. Miller.
Parallel Program Schemata, *J. Comput. Syst. Sci.* 3(2), 147-195.

- a covering tree for ordinary Petri nets
- like supercompilation with lower-node generalization

1993 Alain Finkel.
The minimal coverability graph for Petri nets.
In Grzegorz Rozenberg, ed., *Advances in Petri Nets 1993*, LNCS 674, 210-243.

- an attempt to improve the KM algorithm
- generalization of upper configuration
- a flaw: under-approximation of the minimal coverability set

…and a lot of other works…

2006 Gilles Geeraerts, Jean-François Raskin, and Laurent Van Begin.
Expand, Enlarge and Check: New algorithms for the coverability problem of WSTS. *Journal of Computer and System Sciences*, 72(1), 180-203.